Relations and Functions

Free Response Questions

Q: 1 *f* **is a strictly increasing function while** *g* **is a strictly decreasing function. The range of** *f* **and** *g* **are the same as the codomain of** *f* **and** *g* **respectively.** **[2]**

If (*fog* **) is defined, will (** *fog* **) be an invertible function? Justify your answer.**

Q: 2 ***Prathibha Karanji*** **is an innovative program by the Government of Karnataka, India, where cultural and literacy competitions are held between schools at cluster, block, district and state levels.** **[5]**

One of those competitions - Yogasana, is conducted under two categories - Middle school and High school. From a certain district, three students from middle school and two students from high school were selected for the state level.

Let M = { m_1, m_2, m_3 **}, H = {** h_1, h_2 **}, represent the set of students from middle school and high school respectively who got selected for the state level from that district.**

i) A relation R:M -> M is defined by R = {(x, y **):** x **and** y **are students from the same category}. Show that R is an equivalence relation.**

ii) A function *f* **:M -> H is defined by** *f* **= {(** m_1, h_1 **), (** m_2, h_2 **), (** m_3, h_2 **)} . Show that** *f* **is onto but not one-one.**

Q: 3 **Two functions** *f* **: R -> R and** *g* **: R ->** ***R*****, where R is the set of real numbers, are defined as follows:** **[5]**

$f(x) = (x + \sin x)$
$g(x) = (-x - \cos x)$

i) Find (*fog* **)(** x **).**
ii) Find (*gof* **)(** x **).**
iii) Using i) and ii), show that (*gof* **)(0) - sin(1) = (** *fog* **)(0).**

Show your steps.

Q: 4 **[5]**

If $f(x) = 3 + \left(\frac{e^{3x}+e^{-3x}}{e^{3x}-e^{-3x}}\right)$ and $f^{-1}(x) = \frac{1}{A}g(x)$, find :

i) the value of A.
ii) $g(x)$.

Show your steps.

Q.No	What to look for	Marks
1	Writes that, f and g are one-to-one functions since they are strictly increasing and strictly decreasing respectively.	0.5
	Writes that, (fog) is also one-to-one since f and g are one-to-one.	0.5
	Writes that, (fog) is onto since it is given that f and g are onto functions.	0.5
	Concludes that, (fog) is a bijective function and therefore invertible.	0.5
2	i) Shows that R is reflective. The working may look as follows: x and x are from the same category => $(x,x) \in R$ for every $x \in M$	0.5
	Shows that R is symmetric. The working may look as follows: $(x,y) \in R$ => x and y are from the same category. => y and x are from the same category. => $(y,x) \in R$	1
	Shows that R is transitive. The working may look as follows: $(x,y) \in R$ and $(y,z) \in R$ => x and y are from the same category and, y and z are from the same category. => x,y and z are from the same category. => $(x,z) \in R$	1
	Uses steps 1, 2 and 3 to conclude that R is an equivalence relation.	0.5
	ii) Writes that the Range of $f = \{h_1, h_2\}$ = codomain of f. Hence concludes that f is onto.	1

Q.No	What to look for	Marks
	Writes that $f(m_2) = f(m_3) = h_2$, but $m_2 \neq m_3$. Hence concludes that f is not one-one.	**1**
3	**i) Finds (*fog*)(*x*) as follows:** $f[g(x)] = f(-x - \cos x) = -x - \cos x + \sin(-x - \cos x)$	**1.5**
	ii) Finds (*gof*)(*x*) as follows: $g[f(x)] = g(x + \sin x) = -(x + \sin x) - \cos(x + \sin x)$	**1.5**
	iii) Substitutes $x = 0$ in the expression obtained in step 1 to find (*fog*)(0) as [-1 - sin(1)].	**0.5**
	Substitutes $x = 0$ in the expression obtained in step 2 to find (*gof*)(0) as (-1).	**0.5**
	Finds (*gof*)(0) - sin(1) as [-1 - sin(1)]. **Concludes that (*gof*)(0) - sin(1) = (*fog*)(0).**	**1**
4	**Equates f(x) to y and writes:** $f(x) = y = 3 + \left(\frac{e^{6x}+1}{e^{6x}-1}\right)$	**0.5**
	Simplifies the above equation to get: $y = \frac{4e^{6x}-2}{e^{6x}-1}$	**1**
	Simplifies the above equation to get: $e^{6x} = \frac{y-2}{y-4}$	**1**

Q.No	What to look for	Marks
	Simplifies the above equation to get: $x = f^{-1}(y) = \frac{1}{6}\log_e \frac{y-2}{y-4}$	**1.5**
	Rewrites the above equation in terms of *x* as: $f^{-1}(x) = \frac{1}{6}\log_e \frac{x-2}{x-4}$	**0.5**
	Compares the above equation with the given $f^{-1}(x)$ and writes: $A = 6$ and $g(x) = \log_e\left(\frac{x-2}{x-4}\right)$	**0.5**